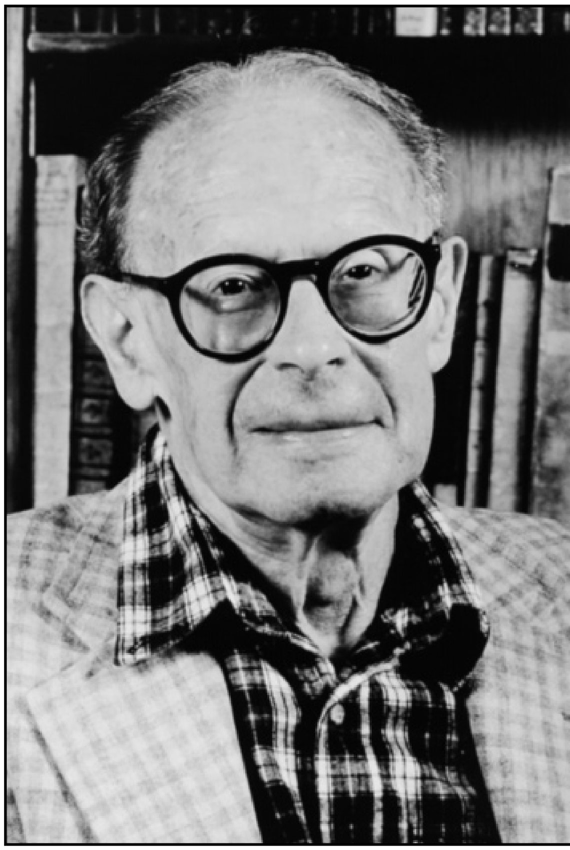
ANDRÉ WEIL



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THE MATHEMATICIAN André Weil died in Princeton at the age of ninety-two, after a gradual decline in his physical, but not mental, capacities. In January 1999, a conference on his work and its influence took place at the Institute for Advanced Study, the poster of which gave this capsule description of him:

> A man of formidable intellectual power, moved by a global view and knowledge of mathematics, of its history and a strong belief in its unity, André Weil has profoundly influenced the course of mathematics by the breadth and depth of his publications, his correspondence and his leading contributions to the work of N. Bourbaki,

a statement of which this text is basically just an elaboration.

In attempting to give an impression of Weil's work and thought processes, I shall have to navigate between the Charybdis of undefined mathematical jargon and the Scylla of vague, seemingly but not necessarily more understandable, statements. I hope the reader will bear with me!

André Weil was born in Paris in a Jewish family, to an Alsatian physician and his Russian-born wife, of Austrian origin. Together with his sister, who was three years younger than he and became the later famous philosopher, he grew up in a stimulating, intellectual atmosphere. He and his sister were very close, challenged one another in many ways, e.g., by reciting by heart long passages of French classics, and were motivated to learn German by their parents' use of it when they did not want to be understood by them. They were both exceptional, knew it, and were not averse to pranks and practical jokes (and wondering on occasion whether all that did not cause them to be viewed as brats). Very early André Weil displayed exceptional gifts for mathematics. He was introduced at fifteen to, and then mentored by, one of the outstanding French mathematicians at the time, Jacques Hadamard. He entered the Ecole Normale Supérieure at sixteen (the usual age is eighteen) and graduated first in his class in 1925. His interests were not confined to mathematics. They ranged deep and wide, including notably Latin, Greek, and Sanskrit and their literatures and cultures, besides classical European literature, art, and music.

The First World War had decimated French youth, and a whole generation of mathematicians had been wiped out, so students had to rely on the previous one, some of whose members were very famous, but André Weil and his friends at the Ecole Normale soon realized that, with the exception of J. Hadamard and Élie Cartan, whom they admired greatly, their professors were mostly out of touch with recent developments, so André Weil decided early on to travel. He visited first Italy, where he got acquainted with functional analysis and algebraic geometry and then, even more important, Germany. There a new generation of mathematicians was developing a flourishing school at the cutting edge, with strong leanings on algebra, or, more accurately, on algebraic methods in various parts of mathematics. Back in Paris, he wrote a thesis on the arithmetic of algebraic curves (1928), proving what was to become known as the Mordell-Weil theorem: for such a curve over a number field, the rational points in its Jacobian form a finitely generated group. It has become a classical result, but generated little interest at the time. Still, it attracted the attention of the German mathematician C. L. Siegel, ten years Weil's senior. This was the beginning of a lifelong relationship, which was to exert a significant influence on both.

Very early on, maybe inspired by J. Hadamard, who had extremely wide interests and was running the only seminar in France devoted to current developments in mathematics, André Weil was striving to acquire a global view of mathematics. A Swiss contemporary mathematician, G. deRham, who spent some time in Paris in the late twenties, once told me that Weil had decided at some point to read all the new papers as they appeared, not necessarily to understand them in detail, but at least to extract the essential ideas. Soon, however, he had to realize this project was overly ambitious and gave it up, but the concern for mathematics as a whole remained a major one.

André Weil did not get at first a satisfactory position in France, so when his professor of Sanskrit (Sylvain Lévi, at the Collège de France) asked whether he would be interested in teaching French civilization at a university in India, he eagerly accepted. It turned out that the position evaporated, but André Weil was asked instead to lecture on mathematics at the University of Aligarh, where he remained for two years (1930–32). He used to the hilt the opportunity to immerse himself in all aspects of India: culture, religion, literature, people, history, scenery, archeology, and so on, traveling all over, often under primitive conditions. In literature, the Bhagavad-Gītā, which he had already read as a student, appears to have been a constant companion in his life, even a source of guidance at some crucial times.

Back in France, he received after some time a position at the University of Strasbourg (1933–39), where he found his old friend Henri Cartan, a son of Élie Cartan. Among their duties was the teaching of differential and integral calculus, for which the standard text in France was the *Traité d'Analyse* of E. Goursat. They found it quite unsatisfactory. Cartan would shower Weil with questions on how to teach this or that point properly, so that, to get it over with, André Weil suggested they write a new *Traité d'Analyse.* This idea was communicated to a few like-minded mathematicians, mostly from the Ecole Normale, who soon agreed to participate. This was the birth of Nicolas Bourbaki, a pen name for a group of French mathematicians. The initial project grew soon to a much more ambitious one, namely, to supply foundations for all of mathematics. This led to the publication of numerous volumes (though not of a *traité d'analyse*) from 1939 on. To achieve his goal, Bourbaki adopted an extremely general and abstract presentation, and was also keen on using a very rigorous style of exposition, at variance with the often flowery, but at times rather vague, style in vogue in France. This, and a natural inclination to practical jokes and to arrogance, did not endear Bourbaki to the mathematical community, and his work was quite controversial in its first years. However, from the late forties on, there was a "French explosion," an avalanche of results, due to a large extent to members of, or people mathematically related to, Bourbaki. Although of course natural talents played an essential role, the pattern and approach to mathematics were sufficiently common to make it clear (as it was to the players themselves) that the Bourbaki methodology was an undeniable underpinning, a powerful help, and that launched Bourbaki as a major influence worldwide. This was not due to Weil alone. It was a common effort in which other participants played an important, even essential, role, too, but there was general agreement in the group that during the first twenty years, until his mandatory retirement at age fifty, Weil had been the driving force, the brain of the enterprise, the only one in the early stages who had the command of mathematics needed to conceive of the plan underlying Bourbaki.

In summer 1939, A. Weil and his wife, Eveline, were in Finland when the Second World War broke out. He decided not to return to France to join the army, though it was his legal obligation, of course. While his wife went back to France, he remained temporarily in Finland, before deciding which course of action to take. He was soon arrested by the Finland police, who accused him of being a Soviet spy and tried to build a case based on totally misinterpreted personal papers. He felt his life was threatened, but was only expelled and, via Sweden and England, eventually delivered to the French authorities, put in prison, and later condemned for "insoumission" (being AWOL), rather than desertion, which might have cost him his life. His conditions in prison, at first somewhat hard, gradually improved: he could communicate with, and occasionally see, his family, had a lively correspondence with his sister, and could receive some books and work. At that time, he proved one of his most famous results, the "Riemann hypothesis for curves over finite fields." (Hearing about it, a mathematical friend, J. Dieudonné, who had at first written to him letters of sympathy, commiserating with his sad situation, changed his tune, envying his being able to work quietly on his mathematics!) Weil was later freed, and managed to travel with Eveline to the U.S., where he spent the rest of the war, on rather meager fellowships or in temporary low-rank positions in universities. After the war, he taught for two years in Brazil at the University of São Paulo, until he was finally offered, and accepted, a professorship commensurate with his talents, at the University of Chicago (1947). From then on, his life was the normal one of an academic of first rank, uneventful in comparison with his early years (only one change of institution: he became professor at the Institute for Advanced Study in 1958, retired to the emeritus status in 1976, and lived in Princeton for the rest of his life), so that he stopped at that point his beautifully written and fascinating autobiography: *Souvenirs d'apprentissage* (The apprenticeship of a mathematician), Birkhäuser 1991 (1992).

Weil could now work under favorable conditions, was at his peak, and published and corresponded widely. Accustomed to be a leader since childhood, in view of his precocious and exceptional gifts, he was a driving force, directly or indirectly influencing many. Truth to tell, being around him was not always that congenial, in view of his sharp, ironical wit (though he did not mind being answered in kind), feisty character, and awesome knowledge, but it was extremely stimulating, since he was always ready, even eager, to discuss mathematics and generous with his insights.

His output offers an extraordinary combination of foundational work, to secure a solid basis in some area, of often decisive contributions at the cutting edge, solving old or new problems, and of forays into unknown territory, in the form of problems or conjectures, guided by a seemingly infallible sense for the directions into which one should forge ahead.

Of course, I feel quite uncomfortable in making such a statement without backing it up in any way, so allow me to turn to the mathematicians to give an idea of these facets of his output in at least one area, algebraic geometry. The theorem he had proved in 1940 (see above) relied on some facts of algebraic geometry for some of which there was no solid reference. Moreover, the development of algebraic geometry, from "classical" (i.e., projective or affine complex varieties) to "abstract" (varieties over arbitrary fields), was also crying out for reliable foundations. It took him several years to supply them in a massive (and rather arid) treatise, *Foundations of algebraic geometry* (1946), the only comprehensive basis for algebraic geometry for a number of years. Although dealing with a very general "abstract situation," he developed it in part in analogy with the theory of differentiable manifolds in differential geometry, and also with some constructions in algebraic topology. It was followed, among other items, by a monograph proving in full his 1940 result, by foundations for Abelian varieties, fiber bundles in algebraic geometry, algebraic groups, the advocacy of the use of analytic fiber bundles in several complex variables, and, in 1949, in a short note, by a series of conjectures (soon called the Weil conjectures), which were to have an enormous impact on algebraic geometry. In particular, he postulated the existence of a cohomology theory in this setup, with properties allowing one to transcribe known arguments in algebraic topology, such as the Lefschetz fixed point theorem, a bold idea, unique to him, way ahead of its time. It was implemented some ten years later by A. Grothendieck (*étale* cohomology), and it took twenty-five years before Deligne proved the last, and by far hardest, of these conjectures, with far-reaching consequences, not yet exhausted.

So far, I have said little of what has arguably been Weil's most abiding interest in mathematics: "Zeta functions." The first one was used by B. Riemann in 1857 to study the distribution of prime numbers among positive integers. The "Riemann hypothesis" about the zeroes of this function is still unproved and is generally viewed as the Holy Grail of mathematics. The introduction of this function to study the discrete (the integers) in a continuous framework (real or complex numbers) was quite revolutionary, and proved to be immensely fruitful. Zeta functions, with corresponding Riemann hypotheses, have proliferated in analysis, algebraic geometry, and number theory, and were always on Weil's mind. (His 1940 theorem dealt with one kind and his 1949 conjectures with generalizations of it.) He was convinced that the problem of the Riemann hypothesis, even in the original case, had to be attacked broadly. How broadly can be explained only in mathematical terms, of course, but he drew an analogy with the Rosetta Stone, which seems to me so typical of his thought processes and of the aesthetic component in his approach to mathematics that I cannot resist trying to give an idea of it, as imprecise as it has to be. It is developed in a short article: "De la métaphysique aux mathématiques" (From metaphysics to mathematics), *Science* 1960: 52–56; *Collected Papers* 2: 406–12.

"Metaphysics," he explains, is meant here in the sense of the eighteenth-century mathematicians, when they spoke of, say, "the metaphysics of the theory of equations": ". . . a collection of vague analogies, difficult to grasp and difficult to formulate, which nevertheless appeared to them to play an important role at certain moments in the research and discovery in mathematics." Then he elaborates:

> Nothing is more fecund, all the mathematicians know it, than those obscure analogies, the blurred reflections from one theory to another . . . nothing gives more pleasure to the researcher. One day the illusion drifts away, the premonition changes to a certitude: the twin theories reveal their common source before disappearing; as the *Gītā* teaches it, knowledge and indifference are reached at the

same time. The metaphysics has become mathematics, ready to form the subject matter of a treatise, the cold beauty of which cannot move us anymore.

He continues: “Fortunately for researchers, as the fogs clear away on some point, they reappear on another. A major part of the Tokyo Colloquium [1955] was devoted to the analogies between number theory and the theory of algebraic functions. There we are still fully in metaphysics. . . .”

“Algebraic functions” alludes here to a theory built up by Riemann by analytical, transcendental means. To link it to number theory, guided by “obscure analogies,” is a problem that had fascinated Weil early on (as already hinted by the title of his thesis), and he felt that progress was still scant by 1960. Meanwhile, a third topic had appeared: “algebraic curves over finite fields” (the subject matter of his 1940 theorem), which was easier to relate to the other two and thus served as an intermediary. These items, and many generalizations or related results, formed an enormous amount of mathematics naturally divided into three parts, each with its own framework (in brief, transcendental, arithmetic, and algebraico-geometric) and techniques. As Weil puts it, we are faced with a text in three parts (he calls them columns), each written in its own language, called by him Riemannian, arithmetic, and Italian respectively, in analogy with the Rosetta Stone. However, there is a huge difference: the latter contains the same text in the three languages (or, rather, assuming this, Champollion was able to decipher Egyptian hieroglyphic writing), while we have here only in each column fragments of what is hoped to be similar texts, once completed.

The task of the mathematicians, then, is to add translations of a given fragment into the other columns, to transform those obscure analogies into mathematics, and eventually to build a dictionary that would allow one to pass from one column to the others. If it were sufficiently complete, then the Riemann hypothesis would be proved, Weil concludes, wondering how long mathematics will have to wait for a Champollion.

As an illustration of his outlook, let me mention a paper ([1972]; *Collected Papers* 3: 249–64), where he formulates a statement in “Riemannian” language, the truth of which would imply that of the Riemann hypothesis (for many zeta functions), points out that it has an analogue in “Italian” that, in view of his earlier work, is a proven theorem, and comments that this provides for him, perhaps, the strongest evidence in favor of the original Riemann hypothesis, one of many examples of his unshakable belief in the unity and harmony of mathematics.

Weil was indeed fluent in the three languages, and many of his works can be interpreted as contributions to the dictionary—but not all. In particular, as befits a man with his cultural interests, he had a strong commitment to the history of mathematics, which culminated in a history of number theory from 1800 B.C. to 1800 A.D. (from Hammurapi to Legendre). Much earlier it had been at the origin of the *Historical Notes* in Bourbaki, to which he was a main contributor until he retired.

As a mathematician, he is shown by his work to be at the same time an architect, a builder, and a poet: an architect for fostering a global view of mathematics and striving to display its fundamental unity, a builder by his specific, often decisive, contributions to a great variety of topics, and a poet by his search for elegance, beauty, and hidden harmonies.

Elected 1995

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